# CHALLENGE IN THE CLASSROOM
# THE METHODS OF R. L. MOORE

(Transcript of video with shot descriptions)

(Music)

(01:00:34:29)
PAN DOWN FROM TOWER AT U.T., AUSTIN

SERIES OF DISSOLVES OF STILLS SHOWING THE U.T. CAMPUS, STUDENTS, STUDENTS IN THE CLASSROOM WITH BLACKBOARD, A STUDENT WORKING AT THE BLACKBOARD, STUDENTS SITTING IN THE CLASSROOM WITH R. L. MOORE SITTING IN THE ROOM.

(01:01:06:16)
MAIN TITLE SUPERIMPOSED:

CHALLENGE IN THE CLASSROOM

(01:01:11:10)
SECOND TITLE FADES IN OVER SHOT OF MOORE FACING CAMERA.

THE METHODS OF R. L. MOORE

(01:01:16:23)
TITLES FADE OUT IN DISSOLVE TO MEDIUM STILL, PROFILE OF MOORE.

(01:01:19:06)
DISSOLVE TO WIDER STILL OF MOORE IN PROFILE, CREDITS APPEAR ON RIGHT SIDE OF SCREEN:

Committee on Educational Media
Mathematical Association of America

(01:01:26:04)
DISSOLVE TO STILL OF MOORE AND R. A. ROSENBAUM WALKING ON CAMPUS.

(01:01:28:13)
DISSOLVE TO STILL OF MOORE AND R. A. ROSENBAUM SITTING, FACING EACH OTHER ON A BENCH.

(01:01:30:27)
CUT TO CLOSE-UP STILL OF ROSENBAUM LOOKING UP INTO CAMERA.

(01:01:32:02)
TITLE FADES IN:

Interviewer:
R. A. Rosenbaum
Wesleyan University

(01:01:40:05)
DISSOLVE TO MEDIUM LIVE ACTION SHOT OF MOORE TEACHING CLASS.

R. L. MOORE

Suppose you think... If you don't see how to do it immediately, suppose you think just a few minutes. And when somebody who feels that he can prove it, as soon as youÆve decided you're ready to do it, why let me know.

(01:01:56:00)

NARRATOR

A class in Austin, Texas, 1964. A very unusual class in many ways. The teacher:

(01:02:04:26)
CUT FROM R. L. MOORE SITTING DOWN TO CLOSE-UP OF HIM SEATED IN THE SAME ROOM.

NARRATOR

Professor R. L. Moore.

(01:02:07:01)
CUT TO WIDE SHOT OF CLASSROOM. A STUDENT RISES AND APPROACHES THE BLACKBOARD. MOORE ADDRESSES HIM.

R. L. MOORE

... Mr. W., you go ahead with it. Go ahead to the board. (LIVE ACTION VOICE FADES DOWN)

(01:02:09:29)
R.L. MOORE INTERVIEW VOICE IS OVER STUDENT APPROACHING BLACKBOARD.

(01:02:15:21)
DISSOLVE TO LIVE SHOT OF INTERVIEW. MOORE FACES CAMERA STANDING BEFORE BLACKBOARD.

R.L. MOORE

Years ago, when I gave a course in advanced Calculus, the course was so different from courses ordinarily given by others with the same title, that I thought it would be better to change the title and the number. About 1941 the title was changed to Introduction to the Foundations of Analysis, and the number was changed to 24, and later on automatically to 624.

Ordinarily in this course, sometimes n... near the beginning of the course, I raise the question, whether or not there exists on the x axis a closed and bounded point set M such that each point of M is a limit point of M, but M contains no interval.

One year... a... a student said that he had proved this theorem. He went to the board and before he'd uttered more than three sentences, I asked him whether he had been reading anything on this subject. He said no, but he had talked to someone about it. I said, "Well that's enough. You've spoiled this for this class," and he sat down. After class, in the hall, another student said, "He certainly did spoil it. After he said that, it was easy to see what the answer was."

Now I suppose some people will say, "Why do you make so much on that simple thing. Why do you... attach so much importance to whether or not a person can

show that there exists such a point set. Doesn't everybody in mathematics know the answer?"

(01:04:46:18) CAMERA ZOOMS IN

Well, I'd like for each person who says that to ask himself how he found out what the answer was. Did he work it out for himself? Or was he not told the answer? And how does he know what difficulties he may have run into if he had tried to settle the question without any hint as to what the answer was.

In this connection I've... often thought of two statements. One of them is a statement by Anatole France that, to know is nothing. To imagine is everything... The other statement is one that I rea... read somewhere in a biographical sketch, I think, of Enrico Fermi. I'm not sure who wrote it; I wish I did. But it went something like this: It is only the clearest of minds, only the clearest of minds, that are the first to think of something which, when once thought of, is clear to everybody. And I wonder if this isn't a good example of just that thing? Cantor thought of this, a lot of other people read about it, (01:06:28:12) BEGIN ZOOM OUT) and it seemed very simple to them. But could they have thought of it themselves in the first place?

(01:06:34:26)
EXTERIOR SHOT OF FOUNTAIN WITH AMBIENT NOISE.

(01:06:38:08)
CUT TO WIDE SHOT OF MOORE AND ROSENBAUM WALKING BY FOUNTAIN.

NARRATOR

R. L. Moore has been teaching others to think for themselves for more than 60 years. And his doctoral students, including many who came from other fields--chemistry, medicine--his doctoral students include many of the best known names in American mathematics.

NARRATION IS OVER LISTS OF NAMES APPEARING ON BLANK CARDS

(01:06:43:27) CARD #1

J. R. Kline, Ph.D. 1916
G. H. Hallett, Ph.D. 1920
Anna M. Mullikin, Ph.D. 1922
R. L. Wilder, Ph.D. 1923

(01:06:51:15) DISSOLVE TO CARD # 2

R. G. Lubben, Ph.D. 1925
G. T. Whyburn, Ph.D. 1927
J. H. Roberts, Ph.D. 1929
C. M. Cleveland, Ph.D. 1930

(01:06:56:11) DISSOLVE TO CARD #3

J. L. Dorroh, Ph.D. 1930
C. W. Vickery, Ph.D. 1932
E. C. Klipple, Ph.D. 1932
R. E. Basye, Ph.D. 1933

(01:07:01:09) DISSOLVE TO CARD #4

F. B. Jones, Ph.D. 1935
H. C. Miller, , Ph.D. 1941
R. H. Sorgenfrey, Ph.D. 1941
R. L. Swain, Ph.D. 1941

(01:07:06:10) DISSOLVE TO CARD #5

Gail S. Young, Ph.D. 1942
R. H. Bing, Ph.D. 1945
E. E. Moise, Ph.D. 1947
R. D. Anderson, Ph.D. 1948

(01:07:11:08) DISSOLVE TO CARD #6

Mary Ellen Estill Rudin, Ph.D. 1949
C. E. Burgess, Ph.D. 1951
Mary-Elizabeth Hamstrom, Ph.D. 1951 or 1952
Eldon Dyer, Ph.D. 1952

(01:07:16:06) DISSOLVE TO CARD #7

B. J. Ball, Ph.D. 1952
J. M. Slye, Ph.D. 1953
J. T. Mohat, Ph.D. 1955
B. J. Pearson, Ph.D. 1955

(01:07:21:10) DISSOLVE TO CARD #8

Wm. S. Mahavier, Ph.D. 1957
Steve Armentrout, Ph.D. 1958
J. N. Younglove, Ph.D. 1958
L. B. Treybig, Ph.D. 1958

NARRATOR

Probably no other mathematician in history...

(01:07:26:12) DISSOLVE TO CARD #9

G. W. Henderson, Ph.D. 1959
J. W. Worrell, Ph.D. 1961
Howard Cook, Ph.D. 1962
J. L. Cornette, Ph.D. 1962

NARRATOR (continues)

... has produced so many distinguished students.

(01:07:33:19) FADE UP CARD #10

W. L. Ayres
H. M. Gehman
N. E. Rutt
Leo Zippin

NARRATOR

Through these, and others who came to Texas to work and learn with Moore, he fathered an entire American school of point set topology.

(01:07:41:09)
DISSOLVE TO SHOT OF MOORE WEARING HAT, CHATTING WITH STUDENTS OUTSIDE, CUTAWAYS TO STUDENTS, AND BACK TO MOORE.

NARRATOR

R.L. Moore's own doctorate was in 1905, and after Chicago, Pennsylvania, and Princeton, he came to Texas in 1920 to stay. His own mathematical research is of the highest distinction. And his influence, both on research and teaching, has been transmitted to his students, to the students of his students, and in turn to their students.

(01:08:13:26)
DISSOLVE FROM MOORE TO CLASSROOM.

STUDENT #1

Now let's see, G* is a subset of some set $\{g_1, g_2, g_3, ...\}$ (FADE DOWN)

(01:08:25:57)

NARRATOR

No lectures here. No books. Inside the classroom or outside. No discussions among the students. They have to work things out, derive theorems from axioms for themselves. They present this work in class at the side board.

(01:08:44:25)
CUT TO MOORE STANDING. HE TAKES A SEAT IN THE CLASSROOM, WATCHING STUDENT AT BOARD, CUT TO STUDENTS WATCHING, AND BACK TO STUDENT #1 WORKING AT BOARD.

NARRATOR

And Moore, from his seat in the corner, observes both the presentation and the class's reaction to it. And if a student does not wish to hear the answer to a problem, he may get up and leave the class.

(01:09:00:06)
CLOSE-UP OF R. L. MOORE FROM INTERVIEW AT BOARD.

R.L. MOORE

Axiom 0 states that every region is a point set. Axiom 1 states that there exists a sequence satisfying certain conditions, which are numbered 1, 2, 3 and 4. Quite a large number of theorems can be derived from Axiom 0 and 1 without use of condition 4. And ordinarily I do not state it until some months have gone by.

(01:09:47:12)
CUT TO MEDIUM SHOT OF MOORE IN SAME INTERVIEW. SHOT CONTINUES WITH ZOOMS IN AND OUT FROM CLOSE TO MEDIUM.

R. L. MOORE

In the session of 1958 to '59, when the time arrived to state condition 4, one member of the class, a Mr. W., did not want to know what this condition was. So the others were told, but he was not.

(01:10:13:20)
CUT TO CLOSE UP OF MOORE IN HALF-PROFILE. HE CONTINUES...

R. L. MOORE

About two years later, in 690, he was standing at the board one day, near the door, explaining something, and I made a remark. He seized the doorknob, flung the door open, and rushed into the hall. He thought that I was about to tell him what condition 4 was. And I suppose some of you think it was ridiculous for Mr. W. to be so determined to remain ignorant concerning condition 4. If so, I do not agree with you at all. In course of time, he thought of a condition, which was such that if it is substituted in place of condition 4, there results an Axiom $1_W$, which is quite interesting and quite different from Axiom 1. I believe that if I had stated condition 4 at the outset, he would never have thought of Axiom $1_W$, or of any of the interesting things he has proved to be true about it.

(01:12:13:07) SLOW ZOOM IN

R. L. MOORE

It has often happened, when I have stated... proposed to a class, some difficult question, and days, weeks, or even months have gone by without anyone in that class settling this question, and finally, the day arrives when some member of the

class asserts that he has a solution, and he goes to the board to give it, then some, in some cases most of the class, leave, will leave the room. Is it a good thing for a student to leave under these circumstances? I think that often it is, but sometimes it is not. And it is quite difficult for me, in the case of some students, to decide whether or not to discourage it.

(01:13:27:22)
CUT TO EXTREME CLOSE-UP PROFILE OF MOORE.

R. L. MOORE

I don't believe I ever said a word to discourage Mr. W. from walking out.

(01:13:38:10)
CUT BACK TO PREVIOUS CLOSE-UP OF MOORE.

R. L. MOORE

I don't believe there's a single instance where he walked out when it would have been better for him if he had stayed in the room.

(01:13:50:23)
CUT TO CLASSROOM WITH MOORE WATCHING STUDENT #1 FROM SEAT AMONG STUDENTS. (01:14:03:07) CUT TO MOORE STANDING IN FRONT OF BOARD.

NARRATOR

Professor Moore does teach differently. But he also has different ideas about what should be taught. Even in Calculus, he prefers to avoid the concept of function, which most other teachers emphasize, and instead he talks about simple graphs.

(01:14:11:04)
DISSOLVE TO MOORE INTERVIEW IN FRONT OF BLACKBOARD.

R. L. MOORE

In 624, and also in my Calculus course, for years, many years, I have defined a simple graph as a point set such that no vertical line contains two points of it.

(01:14:35::23)
CUT TO SHOT OF MOORE WALKING TO HIS RIGHT IN FRONT OF BOARD, STOPPING TO WRITE AND DRAW ON BOARD AS HE SPEAKS, ILLUSTRATING WHAT HE IS SAYING. ZOOM IN ON HIS HAND DRAWING ON BOARD.

R. L. MOORE

Now the statement that the simple graph G is continuous at the point A means that A belongs to G... and if $\alpha$ and $\beta$ are two horizontal lines with A between them, then there exist two vertical lines, H and K with A between them such that every point of G between H and K is between $\alpha$ and $\beta$.

MOORE ERASES FIGURE AND DRAWS ANOTHER.

R. L. MOORE

The statement that *c* is the slope of the simple graph G at the point A, means that *c* is a number such that if *l* is a line containing A with slope *c* , and $\alpha$ is an acute angle with vertex at A and some point of *l* in its interior, then there exist two vertical lines, *h* and *k* with A between them such that every point of G between *h* and *k*, and distinct from A, is in the interior, either of the angle $\alpha$ or the angle vertical to $\alpha$. And furthermore, if *h* and *k* are two vertical lines with A between them, there's some point of G between them and distinct from A.

01:17:16:15) MOORE ERASES BOARD AGAIN AND DRAWS A NEW FIGURE.

R. L. MOORE

Now suppose that M is a point set, regardless of whether it's a simple graph. The line *l* is said to be tangent to the point set M at the point A, if and only if *l* contains A and every circle, the center of A, encloses some point of M distinct from A, and if $\alpha$ is an acute angle with vertex at A and some point of *l* in its interior, then there exists a circle J containing A... enclosing A such that every point of M in the interior of that circle is either... is in the interior either of the angle $\alpha$ or the angle vertical to it.

(01:18:40:08) ZOOM OUT TO MEDIUM SHOT OF MOORE FROM BOARD.

R. L. MOORE

Now I want to emphasize that none of these definitions say anything whatsoever about the notion of a function. The notion of a simple graph is so much simpler than the notion of a function. It's just a point set such that no vertical line contains two points of it. That's much simpler than to say that it's the graph of some function, where a function is what, a collection of two term sequences satisfying certain conditions? The thing is so vague that, I think there's a question as to whether..., how much agreement, if any, there is among mathematicians as to what a function is. But it's very clear what a simple graph is, according to this definition.

Now in this 624 we take up various theorems concerning these notions and relationships between them. (01:19:42:08--MOORE DRAWS NEW FIGURE) For example, is it true that if a point set is closed, its projection onto the x-axis is closed? If a point set is closed, its projection onto the x-axis is closed. Is it true that if a point set has a tangent at the point A, then it has a slope at that point? Under what conditions is it true that a simple graph is continuous? A simple graph whose projection onto the x-axis is a closed and bounded point set, is continuous? Is it true that if a closed point set... Is it true that if a projection onto the x-axis of a.... of a point set is closed, and this point set is closed, then it's continuous? Now I'll give an example of what not to ask. As an example of what not to ask, is this: Is it true that a simple graph is continuous, a simple graph whose projection onto the x-axis is closed, is continuous if and only if that simple graph is both... is both closed and bounded. Why state a thing like that instead of leaving it for the student to think of?

(01:21:40:01)
CUT TO MOORE IN HIS OFFICE BEING INTERVIEWED.

R. A. ROSENBAUM
(Off Screen Voice)

One thing. Sometimes when I was a graduate student; we had a problem--it was considered natural--we'd go out and have a cup of coffee and discuss it.

R. L. MOORE

(groans) Oh no, I think they understand it that I don't want any such thing. I don't want any teamwork.

R. A. ROSENBAUM
(Off Screen Voice)

No teamwork.

R. L. MOORE

In fact, this has often happened, and in the class even. Maybe some student will go to the board to prove a theorem, (BEGIN ZOOM IN ON MOORE) and some student will start to make a suggestion. And I said, "Now, don't make any suggestions. If you find something wrong, call attention to it, but don't try to suggest what…" Suppose he hesitates for awhile, and some student starts to say something. "Don't try to tell him what to do. If he does something wrong, why point it out." Well then suppose that somehow or other the discussion begins to start, and one person suggests something and somebody else suggests something else. And I said, "Now let's don't have any teamwork on this. I... After all this discussion, suppose somebody finally gets a theorem. Whose theorem is it?

R. A. ROSENBAUM
(Off Screen Voice)

...A lot of joint papers have been published, you know.

R. L. MOORE

...I should think he'd want the theorem to be his, not to be some theorem's joint product of a lot of people.

(01:22:49:01)
DISSOLVE TO MOORE AND ROSENBAUM WALKING OUTSIDE. THEY SIT DOWN ON A BENCH.

R. L. MOORE

He said, "I bet that...," and he mentioned this other man, "I bet he doesn't teach that way." That man happened to be somewhere near there. He went up to him and said, "I bet you don't teach that way, do you?" He said, "If I hadn't, I never

would have had any good students." So I guess you didn't ask... ask him that question?

(01:23:14:02)
CUT TO CLOSE-UP OF R. A. ROSENBAUM TALKING TO MOORE.

NARRATOR

Professor R. A. Rosenbaum of Wesleyan University talks with Professor Moore about his philosophy of teaching.

R. A. ROSENBAUM

...concern for the individuals and the students mean a great deal to you, I get the impression that you are inclined to foster competition or rivalry among students. Is this indeed correct?

(01:23:32:23)
CUT TO WIDE SHOT OF MOORE RESPONDING.

R. L. MOORE

That's one of... one of the main features of it, rivalry. To have a class with just one student, oh that's nothing; that isn't the same thing at all. I did on one occasion have a class with one student.

(01:23:46:13)
CUT TO CLOSE-UP OF MOORE TALKING.

R. L. MOORE

...at O. H. Hamilton. I had him by himself in a class, a whole long-session class, and we concentrated on continuous transformations and fixed points. Now he's done work along that line since.

(01:24:02:07)
CUT TO MEDIUM TWO-SHOT OF MOORE AND ROSENBAUM ON BENCH. CUT TO CLOSE-UP OF ROSENBAUM LISTENING AT (01:24:21:28), AND BACK TO TWO-SHOT AT (01:24:27:00), CUT TO ROSENBAUM CLOSE-UP AT (01:25:11:12),

AND BACK TO MOORE AT (01:25:13:13), CUT BACK TO ROSENBAUM LAUGHING AT (01:25:56:06), AND BACK TO MOORE AT (01:25:58:01).

R. L. MOORE

> But ordinarily this goes better if you have a number of students in there--even two or three is rather small--and where there is competition. Now we were talking in the hall there today outside the class with some of these students who were standing around waiting for the class to start, and I was asking some of them a certain question. And I've asked questions like that before in a class. Maybe I'll ask one student, "What will happen if we don't get this [bell starts ringing in background] problem solved during this class? If we don't get it settled during this course, do you want me to tell you the answer?" (ZOOM IN ON MOORE) And if some student says, "Well...," or hesitates about it, and I say, "Well, suppose that we don't get it settled..." Well then maybe some student says, "Well, if it's not settled, I'll go on and continue to work on it." And then I say, "Well, which would you prefer? That someone else should settle it, then that you should not settle it, but someone else would?" Well, sometimes they hesitate as to what to say about that. But one of them said, ahm... "Well, I hope that somebody settles it, whether it's me or not." I said, "You mean that you'd rather have someone else settle it than to not have it settled at all?" And that student said, "Yes." I just wonder what kind of a student that is, whether that student was hypocritical, or whether that student was lacking in real mathematical ability, or just what it was? I try to... I don't think they have much doubt before we get through that I would rather have as an answer, "I hope that I get it and that no one else does...come out with it."

R. A. ROSENBAUM AND MOORE LAUGH. (FADE TO BLACK AT 01:26:09:04)

(01:26:10:14)
CUT TO MOORE IN HIS OFFICE GOING TO SHELF WHERE HE GETS A PILE OF PAPERS FROM UNDER SOME BOOKS.

(01:26:18:08)
CUT TO CLOSER SHOT OF MOORE GETTING PAPERS. WITH THE PAPERS IN HAND HE WALKS BACK TO HIS DESK.

NARRATOR

Professor Moore's convictions about individual work have prompted this collection of articles.

R. L. MOORE

(inaudible)... teamwork. I suppose you understand what I mean. I think in most every one of them there's something said in favor of individual... individual work and against teamwork.

MOORE SITS DOWN.

R. A. ROSENBAUM

And you have quite a collection there.

R. L. MOORE

There's so much talk nowadays about teamwork, in favor of it, (ZOOM IN) but that doesn't mean that everybody feels that way at all. I don't know what this one is going to be, but let's turn to page 296...

(01:26:49:29)
DISSOLVE TO CLOSER SHOT AS MOORE OPENS BOOK.

R. L. MOORE

Oh, this is fine. This is what I want. This is an article by, it's a rather long article, by C. Truesdell. I don't know if any of you know him or not. C. Truesdell. The title of it is, "Recent Advances in Rational Mechanics." On the last page of this, he says, "The work I have described was, has been... The work I have described was done slowly by individuals working alone or, or with a single other individual of like taste. The great teams that produce bombs and vaccines would not have multiplied or deepened the output here." I'm skipping again.

(01:27:54:27)
DISSOLVE TO SHOT OF MOORE AT BOARD, AT (01:28:04:13) CUT TO CLASS WATCHING MOORE.

R. L. MOORE

(still reading in voice over from previous scene)

"This was the work of a handful of men, scattered over a continent and a century, men who were willful, uncompromising, quarrelsome, arrogant and creative."

(01:28:13:19)
CUT TO MOORE TALKING TO STUDENTS IN CLASS

R. L. MOORE

If you didn't follow Mr. Brown's argument, if you didn't follow it at all, and you can work out an argument for yourself, then are you going to be sorry you didn't follow it, Mr. Green?

MR. GREEN (Student)

Yes.

R. L. MOORE

You are?

MR. GREEN (Student)

Well...

R. L. MOORE

I see. If you didn't follow it at all, and if you work out an argument for yourself for next time, are you going to be sorry that you didn't follow it?

MR. GREEN (Student)

No, I didn't. I don't want to follow it if I can work out an argument for myself.

R. L. MOORE

Well, all right, so if you didn't follow it, then work out one for yourself, and then I should think you'd feel good over not having followed it. If you can work out one, that's another matter.

(01:28:51:12)
CUT TO INTERVIEW IN OFFICE. CAMERA IS ON MOORE FROM BEHIND ROSENBAUM.

R. A. ROSENBAUM

Have you ever thought of writing a book, not, a book to try to describe why you... how you teach and how you do it, and why you do it the way you do. Why you sit the way you sit in the classroom and uh... to putting this down in an article or a book?

(01:29:07:01)
CUT TO CLOSE-UP OF MOORE RESPONDING.

R. L. MOORE

(shaking his head in the negative)

No, as a matter of fact, I've... some people have suggested that to me and I just didn't have any sympathy with the idea.

(01:29:17:13)
CUT BACK TO WIDER SHOT WITH ROSENBAUM IN FOREGROUND.

R. L. MOORE

Now this is... Of course, here's a case where I am going on record about these things, but as far as writing a book on it, I wouldn't want to... Well, I wouldn't want to do it even. It would be quite an undertaking for me in the first place to do it. I'd never be satisfied with what I put down there. (ZOOM IN ON MOORE) Look at all the time I've taken just for this. Now I... to write a book, I'd just be too much of a... And when I got through, I don't think I'd ever be satisfied with it.

R. A. ROSENBAUM

You once told me that students would read it and then they'd know what was going on. It would take away their sense of...

R. L. MOORE

Yes, that's it! In fact, I've thought about this in connection with this affair here. Now I don't know how many of them would ever see this film, but I don't know

whether you noticed it or not, but I was pretty careful not to give the solution to that collection Q problem. I was... I didn't want to do it. I didn't want to tell them whether any such collection exists or not.

(01:30:13:16)
CUTAWAY TO MOORE'S HAND.

R. L. MOORE

And I didn't want to tell them how to find out, in case they did do this, how to found out... find out for a particular point set whether it belongs to that collection.

(01:30:21:26)
CUT BACK TO MEDIUM SHOT OF MOORE IN INTERVIEW.

R. L. MOORE

Ah... because I always ask that. In case the collection does exist, can you get some way of finding out for a particular point set, whether it belongs to that collection without actually going through the work or showing directly that it does? Some simple way of telling from the point set, just in a short way of time, whether it belongs to that collection or not? And that's the thing, that's a very difficult thing, and I wouldn't want to state that in this little... (ZOOM OUT)

R. A. ROSENBAUM

...little treatment?

R. L. MOORE

...at all. Someone might see it, and...

(01:30:57:19)
CUT TO CLOSE-UP OF MOORE IN PROFILE IN MID-SENTENCE.

R. L. MOORE

I even hesitated about what I said about connectedness for that reason.

(01:31:03:09)
DISSOLVE TO MOORE STANDING AND WRITING ON BOARD.

R. L. MOORE

Now there's one question that I have always raised in 624 for many years. Does there exist a collection Q of closed and bounded point sets such that there's at least one point set in the collection Q with as many as three points in it? And two, if X is a point set of the collection Q, and T is a continuous transformation of X into some other point set, that point set also belongs to the collection Q. And three, if X and Y are two point sets of the collection Q, and X is non-degenerate, then every image of X... and X is non-degenerate, then there is a continuous transformation throwing X into Y. If X and Y are two point sets of the collection Q, and X is non-degenerate, every point set which is the image of X under a continuous transformation, also belong to Q. Now... (ZOOM IN ) I've often had, and in fact I think ordinarily in my 624 class, someone gets an answer to that question, as to whether such a collection exists. But here's a case where it's possible to do something other than merely to ask for a proof of a theorem. It's true that the question of whether the theorem is true that there's such a collection, but I wonder whether anybody could ever think of that collection without introducing a certain concept. Now how about giving students a chance to introduce concepts as well as prove or disprove theorems? Suppose someone wonders, in fact I've often done that, as to whether or not I might not... ahm... give some kind of theorem which... such that a student could not solve it without introducing the notion of connectedness. Now this year I took particular pains to stay away from the notion of connectedness, never defined it at all, I hint at it, until I raised this question about the collection Q. One student in that class came in one day with a point set like that, just the sum of two mutually exclusive intervals, and said, "That does not belong to Q," and gave a good argument to show that it does not. Then I raised the question of whether she couldn't get something more general than that, and this student did something that surprised me.

MOORE REMOVES A PIECE OF PAPER FROM HIS POCKET.

(01:34:30:12)
CUT TO CLOSE-UP OF HIS HANDS HOLDING THE PIECE OF PAPER AS HE READS IT. SHOT TILTS UP TO MOORE SPEAKING AND PULLS BACK TO BECOME MEDIUM SHOT.

R. L. MOORE

She said, "If there exists a distance between two subsets of M such that there is no point of M in that distance, then M does not belong to Q." She realized that that was not... that was somewhat lacking in clarity, and said, "Now what do I mean by, in that distance?" Well, I suggested that she continue to think about it. And the next meeting of the class, she came in with something that was almost, if not entirely, equivalent of Lennis's definition of a point set that's not connected. Now I would much rather have a definition, a concept, introduced in that way, even... even though--and I'm tempted to say, especially if--it's in a rough form like that at the beginning. If it had been perfect at the beginning, ah... I would wonder whether it wasn't obtained from some other source. So here's an example of when a student thought of a concept that had not been defined, as far as I know, before.

(01:36:01:18)
CUT TO CLASSROOM WITH STUDENT #1 WORKING AT BOARD. MOORE WALKS UP TO HIM AND THEN BACKS AWAY AS THE STUDENT KEEPS WORKING.

R. A. ROSENBAUM

(Voice over classroom scene)

Professor Moore, there's one question that I'd like to ask that stems from my having visited your class and listened to you in action. Some people have said that your (BELLS BEGIN IN BACKGROUND) method of teaching is first rate for students of very high ability, but that it would be very hard on the poor student.

(01:36:32:21)
DISSOLVE TO MEDIUM SHOT OF MOORE AND ROSENBAUM ON THE BENCH. AT (01:37:04:02) CUT TO CLOSE-UP OF ROSENBAUM LISTENING.
CUT BACK TO MOORE AT (01:37:07:06). CUTAWAY TO ROSENBAUM AT (01:37:37:21). BACK TO MOORE AT (01:37:43:03).

R. A. ROSENBAUM

Would you comment on that?

R. L. MOORE

Yes, I'd like to comment on that. Now let me give you an example. Mmm... One summer in my graduate course, I had a student who I believe had studied at either West Point or Annapolis. He was, I believe about 40 years old and apparently he'd got a lot of wrong ideas about mathematics, fundamentally wrong. Wish I could remember more of them. But one example would be something like this. He'd say that... He used the words, "infinitely small." And I said, "What do you mean by infinitely small? How small? Would 1 over 100 be small enough?" "No." "How about 1 over a billion?" "Yes, that would be small enough." Now, that's just a sort of a sample of the kind of ideas that he had. Now if it is true that I teach only for the unusually able students, how can anybody account for the fact that I believe... I don't remember for sure, but I think most... most days during that class I called on him first. And why I called him first? Well I expected him to make some ridiculous statement that I'd want to pursue. Now when I asked him a question, he just said, "I don't know," and never said anything or never asked any questions that I... that would be one thing, but he didn't. Well, now he considered... he continued that way most of the time, in fact, I might say, almost all the time, until one day, ahm... I thought of a certain question to propose to that class, a question that I don't think I'd ever thought of before, and I proposed this. After class, I began to think, well now let's see about solving that thing. And I began to think about it and saw that it was more difficult than I had thought. Well, I finally thought of a certain theorem such that if I proved it, and with the aid of that, I could prove this thing. And then I thought, well I don't think there's much chance that any of them are going to think of that other thing, so I thought none of them will get this the next meeting. Well, next meeting as usual, I called on Mr. H., and he went to the board and he stated that thing that I thought none of them would be capable of thinking of, and proceeded to use it to prove the theorem. Well, after he stated that one thing, I didn't pay much attention to the rest of it because it was too simple. And then another student, and I think one of the best students in the class I thought before, didn't follow his argument, had a hard time trying to follow his argument. Well, but after that, he subsided into his original state. And that continued until the end of the summer term. Then, I suppose it was in September sometime, he came to my office and asked my permission to enter class in 688. Now I don't know just exactly what I said to him, but I imagine that I may have said, "Are you... Are you sure you want to take that course?" (ZOOM IN ) I didn't think that just doing that one good thing was enough to make him

good enough to take 688. Well, he wanted to take it, and he took it. Well, he continued about the same way as before, as that poor performance. Didn't seem like it'd be the same kind of person that could prove that theorem that I mentioned. He continued that way up to a certain time, which is either the last meeting of the class before the December holidays or the last day before the examinations in January. And that day he started on a difficult theorem but didn't finish it. Then he continued it to the next meeting of the class after that, and proved it. And by the end of the second semester he was one of the very best students in the class.

R. A. ROSENBAUM

Is that so?

R. L. MOORE

Now if I hadn't paid any attention to what I considered to be a very poor student, how could you explain that?

(01:41:10:27)
CUT TO MOORE INTERVIEW BEFORE BOARD. HE STARTS FROM STANDING POSITION, PULLS OUT A CHAIR AND SITS WHILE TALKING. SLOW ZOOM IN AS HE SPEAKS.

R. L. MOORE

Some years ago, I had in a class in 688 an outstanding student whom I'll call Mr. S. Seldom, if ever, did he have any occasion to walk out. Seldom, if ever, did any other student present in class a proof of a theorem that Mr. S. had not already proved. Another member of the class said to me, "Being in a class with Mr. S. is like being in a lecture course with Mr. S. as a lecturer, and I do not like lecture courses, so I do not want to take 689 next long session. If another student and I would stay away for the next long session, could we count on your giving 689 in the long session immediately following the next one?" I told him I expected to give it then, and he and the other student stayed out for the whole of the next long session, and came back later according to plan. Now I think that their decision to do this was a wise one under the circumstances. They were both good students, and both went on and got their Ph.D. degrees.

(01:42:56:16)
DISSOLVE TO CLASSROOM WHERE STUDENT # 1 IS STANDING AT BOARD AND MOORE TALKS TO HIM FROM OFF CAMERA BEFORE JOINING HIM AT THE BOARD.

R. L. MOORE

... the green segment... The length of the green segment plus the length of the red segment is equal to what?...

STUDENT #1

Of what? The length of the green one?

MOORE GOES TO BOARD AND REVIEWS WHAT'S THERE.

R. L. MOORE

That's the length of the red one, that's the sum of the lengths of the segments of G and the sum of the lengths of the segments of H, is equal to the length of this whole segment plus the length of what? Of this. That's all I mean.

R. L. MOORE
(Voice Over)

If a person hasn't given a course this way, I think he does not realize (BACKGROUND VOICE: ...green segment ...) how difficult it is for most students to get a bare contradiction of a comparatively simple mathematical statement. It could sound very simple if you say it, but I wish you'd try that on one of the elementary courses and see what you get.

R. L. MOORE

I don't know, if you wanted to prove it?

STUDENT # 1

I didn't want to. No. (Laughter in classroom.)

R. L. MOORE

You just want us to take up the value of N?

STUDENT #1

Yes sir.

R. L. MOORE

Well now, Miss Dingle, can you actually prove that?

(01:44:04:20)
DISSOLVE TO R. A. ROSENBAUM AND R. L. MOORE INTERVIEW ON OUTSIDE BENCH. CUT TO CLOSE-UP OF ROSENBAUM AT (01:44:10:09), CUT BACK TO TWO-SHOT AT (01:44:17:02)

R. A. ROSENBAUM

I'd like to ask something about your own development, if I may? I imagine that you yourself were given books, mathematical books...

R. L. MOORE

Oh, yes, yes. I...

R. A. ROSENBAUM

And uh...

R. L. MOORE

... not only given mathematical books, but well... ah... I went to a private school in Dallas, and to a school with just one teacher, and one... one classroom. He taught shorthand, drawing, Spanish, history, English, algebra and geometry (laughs). And... But I didn't take any Latin there. And I wanted to enter the University of Texas in 1898, and here it was 1897, and... (ZOOM IN ) I became 15 in November of 1897, and I wanted to get a BA degree and Latin was required for the BA degree. So I quit going to that school along about December of 19... 1897, didn't go back anymore, and he lent me some books, lent or gave them to me, I don't know. I have one of them still. One of these books was a book on Calculus by Church. In the preface to that book, the author made some statements about how much time he had spent on this, and hoping that the public would, I don't know what he said exactly, but went on about it as if he was very pleased with the reception of this book, and he'd improved it in various ways. Well, I tried to read that book, and I got to the point where he had something as equal to, say

dx + 2(dx)$^2$, two times squared dx. And he gave an argument to show that it's all right to leave off the (dx)$^2$ because it's so small compared to the dx, that an infinitesimal was a number that goes infinitely many times into 1. And if this number goes infinitely many times into 1, the square of it will go still more times, and so it can, it can just, might as well be discarded. Well, I got so disgusted with that thing, not by talking to anybody else, by just reading it by myself. I didn't discuss it with anybody. I just got so disgusted with it that I wrote here to the university and asked them to send me a copy of the book they used here. When I got that book, and I still have it, have both those books, I still have where I put the date on there, where I wrote in the margin of that book, that is sometime before I came here, 19...1898, or the latter part of 1897, "Oh the difference between this rigorous textbook and that loose treatment of Church." Well, now as a matter of fact since then I decided it wasn't quite as rigorous as I thought it was, but it was so much better than that one of Church.

Now I don't know whether that gives you any idea of how I got started in mathematics. I like mathematics, but that doesn't, that doesn't say why I liked it. And I don't think I could explain that.

R. A. ROSENBAUM

Yeah.

R. L. MOORE

But... I want to say this: how did I study that book? Well, if I had a book here, I could make it a little clearer... Suppose here's...

(01:47:45:28)
CUT TO MOORE'S HANDS LOOKING AT BOOK, COVERING TEXT.

R. L. MOORE

Well, I don't think so... Well, all right. Here's the way I studied. I would read a statement of a theorem. I would read the statement of the theorem, and then I would put a sheet of paper below the statement so as to hide everything under it in the text. I'd look at the statement of the theorem, be very careful to do that and look away until I got that proof of it, and be very careful to hide everything else on that page, below that statement.

(01:48:15:22)
CUT BACK TO MEDIUM SHOT OF MOORE AND ROSENBAUM ON BENCH.

R. L. MOORE

If I thought a figure was necessary, I would draw the figure, then set the book aside and try to prove the theorem without looking at the book anymore. If I did not succeed in doing so without taking too much time, I would again open the book, and be careful to put a sheet of paper just where I had it before, and then move it down very slowly, just one line. And if I didn't get any hint from one line, maybe two lines, until I got some kind of a hint, and then go through the process again. Continue that way until I obtained a proof, which is either partly my proof or the proof in the book, and I think I considered it a failure if I had to follow the whole proof in the book. Now, ahm... I said I used... I indicated I used to do that. What do I do now? I don't think I often if ever read a proof in a mathematical journal to prove a theorem. (SLOW ZOOM IN ON MOORE AT 01:49:37:22) If I read a statement of a theorem in a mathematical journal, and if I'm sufficiently interested in it to try to prove it myself, that's one thing, but I don't usually read proofs of theorems in books or journals. Now one time at the University of Chicago--this might indicate my aversion to this kind of thing--at the University of Chicago at some club meeting, Lennes, a very able man, was reporting on some of his work. He stated a theorem. I listened to that theorem and it sounded interesting. I proceeded to try to prove it. I got a proof and he got through giving a proof. I said, "I think I've got another proof of this." "Boy," he said, "I'd like to see it." I went to the board and gave something, and he said, "That... that was the same thing as I gave."

(01:50:39:02)
CUT BACK TO MEDIUM SHOT OF MOORE AND ROSENBAUM ON BENCH.

R. L. MOORE

Well, I suppose it was. Not having paid attention to him, I... I didn't know firsthand, but if he said it was, I guess it was.

(01:50:46:04)
CUT BACK TO CLOSE-UP OF MOORE.

R. L. MOORE

But that indicates something about my... my own attitude towards reading somebody else's arguments. And now does that answer your...?

R. A. ROSENBAUM

Yes, it does, but now I wonder, do you make an explicit statement to your students
on this issue?

R. L. MOORE

About what?

(01:51:02:29)
CUT TO CLOSE-UP OF ROSENBAUM.

R. A. ROSENBAUM

About uh... reading uh...

R. L. MOORE

Oh, yes.

R. A. ROSENBAUM

...books, and so on? I wondered how you put the...

(01:51:07:17)
CUT TO CLOSE-UP OF MOORE.

R. L. MOORE

Lots. I don't know the exact words, but I don't think that half a week has gone by or even a day has gone by, that there's any misunderstanding on that, except for some very unusual cases. I try to make it terribly clear that they're not supposed to read anything on this subject. I use words like this: "Don't read on this, anything, on this subject,...

(01:51:31:11) CUTAWAY TO MOORE'S HAND OVER PAPERS AS HE SPEAKS.

R. L. MOORE

...except your own notes and what's on the blackboard in this course."

(01:51:36:17) CUT BACK TO INTERVIEW CLOSE-UP OF MOORE.

R. L. MOORE

I make it that strong, and I think I usually do that on the first day so they won't have any... there won't be any misunderstanding about it.

(01:51:45:14)
CUT TO WIDE SHOT OF MOORE AND ROSENBAUM SITTING ON BENCH. THEY RISE AND WALK OUT OF FRAME TO LEFT.

NARRATOR

Perhaps Professor Moore's belief that students learn best by their own efforts comes from his disinclination to follow others, his desire to make his own path.

(01:52:01:27)
DISSOLVE TO WIDE SHOT OF MOORE AND ROSENBAUM WALKING ACROSS THE U. T. CAMPUS IN THE DISTANCE WITH A PAN AND SLOW ZOOM IN ON THEM.

R. L. MOORE
(Voice Over)

Suppose somebody... (UNCLEAR) ...had some forest, and found some interesting things in that forest. He started to go in there and is looking around and sees some animals over here and some birds over here and so forth. Someone takes his hand and says, "You, let me show you," and carries him along through the forest. I'd always think he'd have the feeling that, what... someone took my hand and carried me through this. I'd rather just take some time and find my own way, you know.

(01:52:27:26)
DISSOLVE TO MOORE INTERVIEW AT BOARD.

R. L. MOORE

A foreign mathematician was visiting the United States some years ago and we were talking, and I told him that sometimes when a student gives a long proof and I see a short one, I don't tell him about it. He said, "Why? If you know a short one, you ought to tell him. If you don't, you're not teaching."

(01:53:02:00)
THE MEDIUM SHOT ZOOMS IN FOR A CLOSE-UP. MOORE TURNS AND WALKS ACROSS TO HIS RIGHT, STOPS AND FACES THE CAMERA. ZOOM IN TO HIS FACE.

R. L. MOORE

If you don't, you're not teaching. I'm tempted to paraphrase a well-known statement about government and say, that student is taught the best who is told the least.

(01:53:27:07)
CREDIT MUSIC FADES UP

(01:53:30:18)
DISSOLVE TO STILL SHOT OF MOORE AND ROSENBAUM ON BENCH.
(01:53:32:29) SUPERIMPOSED CREDIT FADES UP:

Produced for the
Panel on Individual Lectures
Primer on Individual Lectures
Committee on Educational Media
Mathematical Association of America

(01:53:41:28)
DISSOLVE TO STILL OF STUDENT AT BOARD IN CLASSROOM.
SUPERIMPOSED CREDIT:

With Financial Support from the NATIONAL SCIENCE FOUNDATION

(01:53:47:10)
DISSOLVE TO WIDER SHOT OF CLASSROOM WITH ONE STUDENT AT BOARD.
SUPERIMPOSED CREDIT:

The cooperation and assistance of the students, staff, and faculty of the University of Texas in the production of this film is gratefully acknowledged.

(01:53:57:24)
DISSOLVE TO STILL OF MOORE STANDING WITH ANOTHER MAN.
SUPERIMPOSED CREDIT:

Produced by
Davidson Films
San Francisco

(01:54:02:11)
DISSOLVE TO STILL OF FILM CREW MEMBER WITH LIGHT STANDS.
SUPERIMPOSED CREDIT:

Executive Production:
A. N. Feldzamen
P. E. Miles

Narration:
Professor Marvin Simbrom
Northwestern University

(01:54:11:01)
DISSOLVE TO U.T. TOWER.
SUPERIMPOSED CREDIT:

Photographed in
Austin, Texas

(01:54:17:14) FADE OUT PICTURE AND MUSIC.

-END-