The Moore Method: What Discovery Learning Is and How It works

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Some millions of years ago a creature with a stick solved the problem of what to eat for lunch by knocking fruit from a tree. At one stroke our ancestor demonstrated problem solving, tool use, and discovery learning.

Discovery Learning

The essentials of discovery learning are: Motivation, Discovery, and Presentation. Hunger, curiosity, and the delight of play supplied the Motivation for our primitive discoverer. Like our primitive ancestor, today's students will make their discoveries, without our having to tell them how they should do this. Presentation? Today students may give their results orally, in writing, or over the Internet, but the idea predates recorded history.

R. L. Moore developed his approach to discovery learning from 1920 to 1969 at the University of Texas. Known as "the Moore Method," his approach was based on lists of axioms, questions to resolve, and results to prove. These sequences challenged students and led them to discover the essentials of the subject. The idea is simple and astonishingly effective. Yet success is critically dependent on the way the class is run, not simply on the theorem sequences.

Moore on Video

To see how this method works, you can watch Moore himself in *Challenge in the Classroom*, a video available from the MAA. To read about the method, look at Chapter 12, "How to Teach," in Paul R. Halmos's autobiography, *I Want To Be A Mathematician*, also from the MAA, or see Halmos's article, "The Problem of Learning to Teach: The Teaching of Problem Solving" from the *American Mathematical Monthly*, May 1975. Another good source is the material on the R. L. Moore Legacy web site at the University of Texas (www.discovery.utexas.edu). I also recommend Keith Devlin's "Devlin's Angle" columns on the MAA website (www.maa.org) for May and June 1999 as a nice source of information about R. L. Moore and his teaching.

The Motivation in Moore's system was provided partly by the student's curiosity and sense of intellectual play. Good-natured competition was also a motivation. Capping everything for Moore's students was his great respect for them and for their ideas. This led to a sense of common intellectual striving that served his students well in their careers.

Robert Lee Moore at his desk in 1935

Martin Ettlinger, who took an MA with Moore before going on to graduate study at Harvard and a distinguished career as a plant products chemist, recently described the atmosphere in Moore's classes as extraordinary. Every student's ideas were listened to carefully and critically. No sniping or discourtesy was tolerated, but every idea was tested before being accepted. Ettlinger said the only other place he encountered this atmosphere was as a Junior Fellow of Harvard's Society of Fellows.

The Discovery in Moore's classes took place mainly outside the classroom, while Presentation lay at the heart of the classroom experience. Moore would ask one of the students whether he or she could present the next item at the board. If the answer was "Yes, sir," the student became the lecturer. Fellow students formed an interested and critical audience. The experience of seeing your dream proof collapse under careful examination by your fellows might be chastening, but the success of a difficult matter disposed of nicely was gratifying.

Moore's Students

Moore's students learned why one needs to check one's own work carefully, and they learned to present their results clearly. They did this for themselves and their fellow students, not simply to please their teacher. Moore used Presentation as a tool to build students' abilities to monitor and improve their own work and to give them the confidence to stand up and present their ideas. These qualities are essential in all walks of life. Moore used careful hints to edge the class forward over difficult material.

He would call on students who had contributed least first, to give them a chance to make a contribution. Moore never missed a chance to praise students for their accomplishments, directly or through others. He actively recruited able students and worked tirelessly to bring out the best in all of his students. All these things and more were part of his art, but the idea behind his method is simple and as applicable now as ever:

Motivate what is to be done. Let the students Discover how to do it. Have the students Present their results in good order before a critical but friendly audience.

Moore was a leader in research and teaching. He and his colleagues H. J. Ettlinger and H. S. Wall directed the work of 139 Ph.D. students. Many of the 50 who earned their degrees with Moore were prolific researchers and teachers including: R. D. Anderson, R H Bing, E. E. Moise, M. E. Rudin, G. T. Whyburn, R. L. Wilder. There were 1034 descendants of this growing line listed on W. T. Mahavier's web site as this was written. See http://math.nich.edu/ted/tree.html.

The Moore method was spread by descendants of the Texas School and picked up by others. John Milnor has praised the Moore-method topology course that he took at Princeton; his instructor, Ralph Fox was not a descendant of the Texas School.

Texas topologists have made great contributions in areas ranging from the topology of arcs and continuua to infinite dimensional spaces and beyond. Though there were no Moore descendants at hand when I was in graduate school, my work on the contractibility of the homomor-

phism group of the Hilbert cube took off from ideas of a Moore descendant, Raymond Y. T. Wong. The Texas school remains a pervasive and productive force.

What is surprising is that Moore's teaching methods have not been more widely studied and used. This may be changing with increased emphasis on student discovery methods. People outside the Moore tradition are beginning to experiment with what the method can do today.

The Modified Moore Method: Inquiry-Based Learning in Mathematics Today

Anyone who has written a thesis understands the demands and rewards of inquiry-based learning. The creative struggle of problem solving and the need to tie things down clearly for others were central to Moore's method of teaching as well—as was the sort of mentoring relationship that a student might have with her Ph.D. advisor. Graduate school provides models for modified Moore-method teaching that we can all recognize.

Students learn to crawl, then walk, then run. Most classes contain some discovery elements, be they challenge problems or projects for individuals or groups. There are many possibilities for inquiry-based learning. The goal is to enable your students to learn more by discovering more for themselves. Michael Martin of Denver University commented that the inquiry-based courses described here provide good background for undergraduate seminars or journal study programs.

John Neuberger of the University of North Texas teaches an analysis course today whose content and teaching style follow lines completely familiar to Moore. But when John teaches numerical analysis or differential equations, he adapts Moore's methods to a world where examples may be worked by students using computers. Here the adaptations concern the content of the course and the tools the students use. You might ask your students to work in teams, each team making its own report. This would be a further modification of the Moore method. For information about John's approaches you may e-mail him (jwn@unt.edu).

The April, 1999 conference at the University of Texas, Austin, on the legacy of R. L. Moore had a session that suggested the range of Moore method courses today. Margaret Symington spoke about the two very different courses she was teaching. We heard from three students: the first was taking Symington's geometry course for future teachers, the second was taking a topology course from the Symington course plus a Moore-method course taught by Haskell Rosenthal on analysis, and the third had taken the topology course the previous semester when it was taught by Michael Starbird, who chaired the session.

Starbird's topology notes formed the basis for the Moore-style topology course Symington was teaching. It is noteworthy that Symington and Rosenthal were not raised in the Moore tradition. They are converts attracted to the method for its teaching power, and these were the first Moore-style courses they had taught.

The previous summer Symington had attended an NSF-funded workshop given by David Henderson on innovative discovery methods for teaching geometry. This workshop was the basis for Symington's course aimed at future teachers. Henderson was a student of R H Bing, who took his doctorate with Moore. Yet Henderson's approach emphasizes geometric intuition rather than axioms. He presents students with geometric situations they can explore and analyze. For example, if you are putting tape onto a curved surface such as a cone or sphere what sorts of paths does it follow?

Everybody who has bandaged a finger knows that there is something interesting here. The students then develop a mathematical description of what is going on and why.

For more information check Henderson's web page (http://math.cornell.edu/~dwh) or e-mail him (dwh@math.cornell.edu). For written material see his two texts, *Experiencing Geometry on Plane* and *Sphere and Differential Geometry: A Geometric Introduction*. Information on Symington's course can be found at http://rene.ma.utexas.edu/users/msyming/geometry/M333L-info.html.

Rosenthal's course was aimed at building a basic understanding of ideas about function spaces. He has been writing up his notes, a large job with much still to be done. Rosenthal commented later: [These notes] already contain quite a bit of material, more in depth than breadth." Depth of coverage is a strength of the method. Breadth can be achieved by writing up more material than would be covered in any one class and varying the areas covered from class to class, this will allow students who have the notes to cover other areas on their own, something the method trains them to do.

What I learned at the April conference and from exchanges with Henderson, Neuberger, Symington, and others is that there are many approaches to the Moore method and that there are materials being developed to help others begin to use the method and adapt it to their needs. Moreover, this work is being taken up by people outside of the Moore tradition who see the method as offering solutions to current problems.

Teachers such as Judy Kennedy at the University of Delaware see Moore-style courses as ideal as introduction to rigorous independent thinking and the use of proofs. These transition courses are becoming common at the junior level. At the University of Texas, Austin the department has asked Michael Starbird to develop such a transition course in number theory for use in Fall of 1999.

Starbird is doing this in collaboration with Edward Burger of Williams College. What is notable about the Burger/Starbird effort is that they plan to provide a full set of supporting materials so that teachers unfamiliar with the Moore method can teach this course. For information about this contact Starbird at starbird@mail.utexas.edu.

Inquiry-based learning is basic in education. We have all experienced it and all teachers use it, to a degree. Is it reasonable to hope that every mathematics department might offer its students at least one modified Moore method course? Given what the method has achieved, I would hope so. ■