My Experiences with the Moore Method

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Without having experienced the Moore Method, would I have become a mathematician? I don't know. I was a very unusual student when I entered Louis McAuley's beginning graduate topology class in the fall of 1965. For starting mathematics at that time I had both handicaps and strengths. I believe the special way of teaching suited my needs particularly well.

There were several ways in which I was different from most other students:

1. Woman
2. Older—36 when I began graduate mathematics and 37 when I joined that class
3. No degree in mathematics—rather, B. A. and M. S. in physics
4. The only mathematics courses I took between ages 20 and 36 were ones meant for teaching high school.
5. In my physics curriculum I had taken only those math courses that were considered important for physics in the 1940's. I had not even become acquainted with the notion of a set until my son brought it home from grade school.
6. In view of all of the above, I received very poor advice from my assigned faculty "adviser." He seemed to think that I was there for my amusement and to hope that I would be gone in a semester. Better initial course selections might have saved me a year and a great deal of frustration in complex variable and abstract algebra.

On the other hand, I had exhibited some abilities.

- I had been judged to be one of the top among those who went to Washington in the Westinghouse Science Talent Search in my senior year of high school.
- My freshman calculus teacher, Dorothy Bernstein (later president of the M.A.A.), very strongly encouraged me. After I told her of my brother's great ability in mathematics she said, "I absolutely refuse to believe that you have a brother who is a better mathematician than you are."
- I completed a bachelor's in physics from the University of Rochester with High Distinction and Phi Beta Kappa in three years. This included all the junior and senior physics, chemistry, and math compressed into one year.
- I was an outstanding graduate student in physics, though I did not complete the Ph.D. (When I had done most of the work other than the physics thesis, my doctoral advisor had problems with the McCarthy Committee of Congress and chose to permanently leave the U. S. A.. About the same time his faculty research partner in the cosmic ray work died suddenly of lung cancer. I would have needed to change to a different area of research and instead just wrote what I had for a master's degree and soon was staying home caring for young children.)

Now consider my introduction to topology through the Moore Method. Compared with standard teaching, it fosters much closer instructor attention to the work of each individual student. In my case, it caused my different approaches to problems to be noticed much more than a standard method would have.

Professor McAuley's presentation started with asking the class to conjecture about the image of a 1-1 function from the real line into the plane. Of course, he listed those conjectures on the board and then our task for the next few weeks was to prove or disprove them, and to make further conjectures. He introduced standard definitions frequently. This was a year course. For those planning theses in topology, there were further courses in point-set and geometric topology. Other faculty members taught these and, following the Moore tradition, reading of books or papers was forbidden. Algebraic topology was a standard course. Seminars were devoted to reading published research papers.

I have the sort of mind that prefers to puzzle through a problem rather than to memorize a standard solution. That kind of personal discovery is a major goal of the Moore Method, as I understand it. It puts a higher priority on a new clumsy method over the repetition of the beautiful old standard solution. Professor McAuley mixed a few major unsolved problems in with the standard ones he suggested to the class—he hoped that someone would have an innovative approach to one. None of us succeeded with those, but there were some very different attacks on other problems. Mine were more different than most, because I was much less familiar with the standard solutions that might have been seen in an undergraduate course or in an advanced analysis course.

The Moore Method stresses building and reinforcing the student's ego. That fit me. I already had that "yes, I can do it" feeling. My father was a chiropractor and, as such, believed that he could do things that even the leading medical doctors could not. To him, no person was an authority about the correctness of any idea. He carried this into other areas, such as politics and religion. I absorbed that lack of respect for standard explanations. I think that attitude contributed to my willingness to attack problems that were unsolved by the leaders in my field. Likely my brother, E. T. Parker, was similarly influenced. He was one of three credited with the refutation of a long-standing Euler conjecture. Incidentally, he took one or two courses under Moore.

Looking back now I see that, even in high school, I leaned toward the self-discovery of mathematics solutions in preference to learning the standard approaches. In beginning algebra my teacher loaned me his college analytical geometry book after I discovered the idea of slopes. I read only the first few chapters. My first experience trying to solve a difficult mathematics problem came in high school geometry. As a challenge we were given the theorem: if two angle bisectors of a triangle are equal, then the triangle is isosceles. The proof of this is far more difficult than its converse. We were told that anyone who succeeded in solving the problem would receive an automatic A for the year course. I found I really enjoyed working on it and probably spent at least a hundred hours puzzling over it. It was not for the grade that I was trying, but rather out of my sheer enjoyment of the effort. In the spring of that year E. T. talked to some college students and learned a solution using inequalities. Of course, he showed me and I found my own different proof using inequalities. I ended this experience by telling the teacher both proofs and the source of the first. (Happily, he was not faced with any decision about my grade.) My point in reciting these high school experiences: They show that I was wishing to do individual thinking about a problem with no help of a

book; my basic thinking was pointed toward something more like the Moore Method than like standard book study.

If we hypothesize that I would not have made it to receive a Ph.D. in mathematics without the Moore Method exposure, then the obvious next question is whether I should have had that degree and the corresponding career. I spent almost 19 years on the faculty of Trenton State College (now The College of New Jersey) teaching more than my share of the advanced courses. I did not apply to a more prestigious institution, because employment at such would have forced my husband and me to live separately. At Trenton I was quite isolated from other research mathematicians, because there were then no colleagues who were actively doing research and, on top of a heavy teaching load, I did not have much time to go to Princeton regularly. In spite of these handicaps, I published more than a dozen papers during my fairly short career. Two were in the Transactions of the American Mathematical Society, and one in each of Fundamenta Mathematica, and the Pacific Journal of Mathematics. Several papers were in various conference proceedings. I was a visitor at the Institute for Advanced Study from June '79 to August '80, and held a grant from NSF during that period. I think these accomplishments show that it was right that I received the doctoral degree. The Moore Method helped me earn it.

All the above was written before I read *R. L. Moore* by John Parker. I suppose that writing before reading is exactly what a person taught by Moore Method would often choose to do. I now turn to my comments based on reading the book:

Some comparisons between Moore's teaching methods and what I saw of McAuley's:

Class size. Since all Rutgers mathematics graduate students were required to take the first year topology course, McAuley's class was much larger than a typical graduate Moore one. When I took it in '65-'66 I think we had 30 to 35 in the fall; he divided us into two groups for the second semester (with this counting as one for his course load). Possibly he did some picking of which people were in which section—I had the impression that the other might have had more of the abler students.

Heterogeneity. McAuley had no chance to pick his students for that first graduate class. His encouragement of certain people must have resulted in some students selection for later courses.

Help outside the classroom. McAuley made himself very available to students for discussion of the class work outside the classroom. The first day he told us that he would welcome phone calls as late as midnight. He also encouraged people to present ideas to him in his office. (I was so isolated from other students that I really have no good idea how many took advantage of these.) These two modes of outside help had several effects

- There was less embarrassment if one were wrong. Particularly for the mediocre student, this made it easier to take part.

- It made more chance for guidance of individuals than could have been done in the large group. He could challenge the best students during conferences.
- It meant that the mediocre students were not having their egos destroyed.

Cooperation between students

I do not recall any statement concerning students talking to one another about the problems in that first course. We may have been told not to. I might not now remember that, because I had almost no opportunity to talk with others anyway. When we were paper-reading in later years, reading together was not forbidden. Looking back on my total career, I would have been better off if I had learned to work more closely with others. My being a woman always hindered such cooperation.

Breadth of mathematical knowledge

For later when I was teaching, I wish I had had a wider background in standard theorems by names. That is more important for teaching in a lesser college. There "power" to develop new theorems is not so highly honored as it is in a truly research institution. McAuley did suggest reading a standard basic text after the first year—at least, to me.

Finding thesis topics

One way McAuley moved us toward thesis subjects was by guiding our reading of original papers. He chose ones that he thought might have lead to further work. This is quite different from carrying the Moore Method all the way to where the student solves an unsolved problem, maybe without even knowing that it is unsolved. It is good that we were taught in seminar how to read papers.

All in all, I think that McAuley chose good ways to modify the Moore Method for the situation that existed at Rutgers. I greatly appreciate his efforts for all of us.